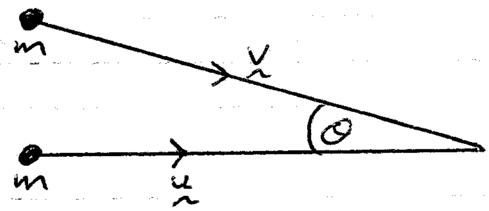
Mechanics Examples Sheet 7 – Solutions

1. $\underset{\sim}{u} = (u, 0)$

$\underset{\sim}{v} = (v\cos\theta, v\sin\theta)$



Conservation of momentum requires

$$m_1 \underset{\sim}{u} + m_2 \underset{\sim}{v} = (m_1 + m_2)\underset{\sim}{w}$$

where, $\underset{\sim}{w}$ is the velocity of the coalesced particle. Thus,

$$m(\underset{\sim}{u} + \underset{\sim}{v}) = 2m\underset{\sim}{w}$$

$$\Rightarrow \underset{\sim}{w} = \frac{1}{2}(u + v\cos\theta, v\sin\theta).$$

Thus, the speed is

$$|\underset{\sim}{w}| = \frac{1}{2}\sqrt{(u+v\cos\theta)^2 + (v\sin\theta)^2}$$

$$= \underline{\underline{\frac{1}{2}\sqrt{u^2+v^2+2uv\cos\theta}}}$$

Conservation of K.E. requires

$$\frac{1}{2}m_1 \underset{\sim}{u}^2 + \frac{1}{2}m_2\underset{\sim}{v}^2 = \frac{1}{2}(m_1+m_2)\underset{\sim}{w}^2.$$

Loss in K.E. is

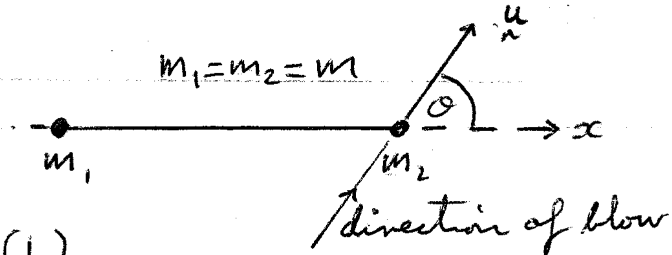
$$\Delta \text{K.E.} = \frac{1}{2}\left[m(\underset{\sim}{u}\cdot\underset{\sim}{u} + \underset{\sim}{v}\cdot\underset{\sim}{v}) - 2m\underset{\sim}{w}\cdot\underset{\sim}{w}\right]$$

$$= \frac{m}{2}\left[u^2+v^2 - \frac{1}{2}(u^2+v^2+2uv\cos\theta)\right]$$

$$= \underline{\underline{\frac{m}{4}(u^2+v^2-2uv\cos\theta)}}$$

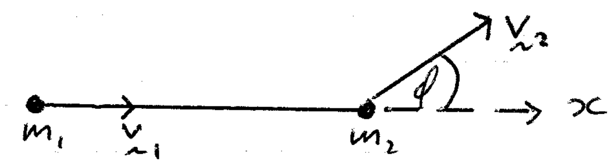
2. Impulse $= m\underset{\sim}{u}$

$$= \underline{\underline{mu(\cos\theta, \sin\theta)}} \quad (1)$$



$\underset{\sim}{V}_2 = v(\cos\phi, \sin\phi)$

$\underset{\sim}{V}_1 = v(\cos\phi, 0)$



Momentum of m_2: $\quad m_2\underset{\sim}{V}_2 = mv(\cos\phi, \sin\phi)$

Momentum of m_1: $\quad m_1\underset{\sim}{V}_1 = mv(\cos\phi, 0)$

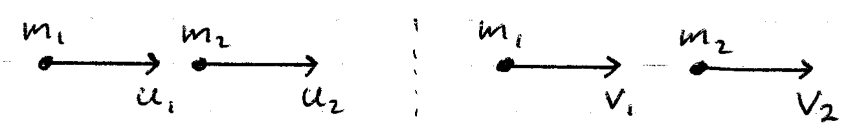
(Remember that the rod is rigid so the particle's initial momentum is necessarily in the direction of the rod.)

Thus, the total momentum is $mv(2\cos\phi, \sin\phi)$ — (2)

If the total momentum is conserved then comparing components of (1) & (2):

$$\left.\begin{aligned} u\cos\theta &= 2\cos\phi \\ u\sin\theta &= \sin\phi \end{aligned}\right\} \Rightarrow \underline{\underline{\tan\phi = 2\tan\theta}}$$

3.



Before collision After collision

From conservation of momentum:

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2 \quad — (*)$$

From Newton's law of restitution:

$$V_2 - V_1 = -(u_2 - u_1), \quad (e = 1) \quad — (1)$$

$$\Rightarrow m_2V_1 = m_2u_2 - m_2u_1 + m_2V_2$$

$$\Rightarrow (m_1 + m_2)V_1 = (m_1 - m_2)u_1 + 2m_2u_2$$

$$\Rightarrow \underline{\underline{V_1 = [(m_1 - m_2)u_1 + 2m_2u_2]/(m_1 + m_2)}} \quad — (2)$$

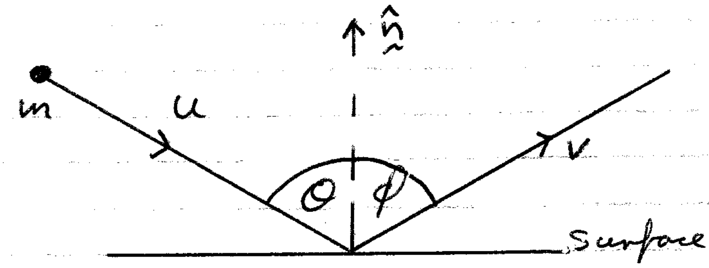
If $m_1 = m_2 = m$ then from (2)

$$V_1 = \frac{2mu_2}{2m} = u_2 \equiv w.$$

From (1) $V_2 - w = -w + u_1 \Rightarrow V_2 = u_1$.

Hence, KE is clearly conserved.

4.



Momentum is conserved ↑↑ to the surface. Hence,

$$u \sin\theta = v \sin\phi. \qquad -(1)$$

We can use Newton's law of restitution for the velocities of the particle ⊥ to the surface. Hence,

$$-(-v\cos\phi) = eu\cos\theta$$

or

$$eu\cos\theta = v\cos\phi \qquad -(2)$$

On squaring (1) & (2), we can eliminate ϕ:

$$e^2u^2\cos^2\theta = v^2 - u^2\sin^2\theta$$

or

$$\underline{\underline{e = \sqrt{\frac{v^2 - u^2\sin^2\theta}{u^2\cos^2\theta}}}}$$

The surface experiences an impulse, I, in the direction of the unit normal, $\hat{n}$. Thus,

$$I = \Delta(\text{momentum})$$

$$= mv\cos\phi - (-mu\cos\ )$$

$$= meu\cos\theta + mu\cos\theta, \quad \text{from (2)}$$

$$= \underline{\underline{mu(1+e)\cos\theta}}$$